

# RECUPERO

## LE ESPRESSIONI CON LE FRAZIONI ALGEBRICHE

### 1 COMPLETA

Semplifica la seguente espressione:

$$\left(x + 1 + \frac{2x + 2}{x - 1}\right) \cdot \left(\frac{x}{x^2 + x}\right)$$

$$\begin{aligned} &\left(x + 1 + \frac{2x + 2}{x - 1}\right) \cdot \left(\frac{x}{x^2 + x}\right) = \\ &= \left(\frac{x + 1}{\dots} + \frac{2x + 2}{x - 1}\right) \cdot \frac{x}{x(\dots + 1)} = \end{aligned}$$

Scomponi il denominatore  $x^2 + x$  e scrivi  $x + 1$  come frazione.

Campo di esistenza:

Determina le C.E. delle frazioni che compaiono.

$$x - 1 \neq 0 \rightarrow x \neq \dots; \quad x \neq 0; \quad x + 1 \neq 0 \rightarrow x \neq \dots$$

$$= \left[ \frac{(x + 1)(\dots) + 2x + \dots}{x - 1} \right] \cdot \frac{x}{x(\dots + 1)} =$$

$$= \frac{x^2 - \dots + 2x + \dots}{x - 1} \cdot \frac{x}{x(\dots + 1)} =$$

Calcola i prodotti indicati ed elimina la parentesi tonda.

$$= \frac{x^2 + 2x + \dots}{x - 1} \cdot \frac{x}{x(\dots + 1)} =$$

Somma i termini simili nella prima frazione.

$$= \frac{(x + \dots) \cdot \cancel{x}}{x - 1} \cdot \frac{\cancel{x}}{\cancel{x}(\dots + 1)} = \frac{x + \dots}{x - 1}$$

Scomponi  $x^2 + 2x + 1$  e semplifica i numeratori con i denominatori.

### 2 PROVA TU

Semplifica la seguente espressione:

$$\frac{a}{a + 1} + \frac{1}{a - 1} + \frac{2a}{a^2 - 1}$$

$$\frac{a}{a + 1} + \frac{1}{a - 1} + \frac{2a}{a^2 - 1} =$$

$$= \frac{a^2 \cdot \cancel{a} + \dots + 1 + 2a}{(a + 1)(\dots - 1)} =$$

$$= \frac{a}{a + 1} + \frac{1}{a - 1} + \frac{2a}{(a + \dots)(a - \dots)} =$$

$$= \frac{a^2 + 2a + 1}{(a + 1)(\dots - 1)} =$$

C.E.:

$$= \frac{(a + \dots) \cdot \cancel{a}}{(a + 1)(\dots - 1)} =$$

$$a + 1 \neq \dots \rightarrow a \neq \dots$$

$$a - \dots \neq 0 \rightarrow a \neq \dots$$

$$= \frac{a + \dots}{\dots - 1}$$

$$= \frac{a(a \dots) + 1(a \dots 1) + 2a}{(a + 1)(\dots - 1)} =$$

## 3 PROVA TU

Semplifica la seguente espressione:

$$\left(x - 2 - \frac{5}{x+2}\right) \cdot \frac{x^2 - 4}{x - 3}$$

$$\begin{aligned} \left(x - 2 - \frac{5}{x+2}\right) \cdot \frac{x^2 - 4}{x - 3} &= \\ &= \left(\frac{x-2}{1} - \frac{5}{x+2}\right) \cdot \frac{(x+\dots)(x-\dots)}{x-3} = \end{aligned}$$

C.E.:

$$x + \dots \neq 0 \rightarrow x \neq \dots$$

$$x - 3 \neq 0 \rightarrow x \neq \dots$$

$$\begin{aligned} &= \left[ \frac{(x-2)(x+\dots) - 5}{x+2} \right] \cdot \frac{(x+\dots)(x-\dots)}{x-3} = \\ &= \frac{x^2 - \dots - 5}{x+2} \cdot \frac{(x+\dots)(x-\dots)}{x-3} = \\ &= \frac{x^2 - \dots}{x+2} \cdot \frac{(x+\dots)(x-\dots)}{x-3} = \\ &= \frac{(x+\dots)(x-\dots)}{\cancel{x+2}} \cdot \frac{(x+\dots)(x-\dots)}{\cancel{x-3}} = \\ &= (x+\dots)(x-\dots). \end{aligned}$$

## 4 PROVA TU

Semplifica la seguente espressione:

$$\left(\frac{4x^2 - 4x + 1}{y^2 - 1}\right)^2 \cdot (2x^2 - x)^{-2} \cdot (y^3 - 1)^3$$

$$\left(\frac{4x^2 - 4x + 1}{y^2 - 1}\right)^2 \cdot (2x^2 - x)^{-2} \cdot (y^3 - 1)^3 =$$

$$\begin{aligned} &\left[ \frac{(2x-1)\dots}{(y-1)(y+\dots)} \right]^{\dots} \cdot \frac{1}{(2x^2 - \dots)^{\dots}} \cdot [(y - \dots)(y^2 + y + 1)]^{\dots} = \\ &= \left[ \frac{(2x-1)\dots}{(y-1)(y+\dots)} \right]^{\dots} \cdot \frac{1}{x \dots (2x-1)^{\dots}} \cdot (y - \dots)^{\dots} (y^2 + y + 1)^{\dots} = \end{aligned}$$

$$\text{C.E.: } y \neq \pm \dots \wedge x \neq \dots \wedge x \neq \frac{1}{\dots}$$

$$\begin{aligned} &= \frac{(2x-1)\dots}{(\cancel{y-1})\dots(y+\dots)^{\dots}} \cdot \frac{1}{x \dots (\cancel{2x-1})\dots} \cdot (y - \dots)^{\dots} (y^2 + y + 1)^{\dots} = \\ &= \frac{(2x-1)\dots (y-1)(y^2 + y + 1)^{\dots}}{x \dots (y+\dots)^{\dots}} \end{aligned}$$

Semplifica le seguenti espressioni.

- 5**  $\frac{1}{4b} - \frac{2}{3b} + \frac{1}{12b}$   $\left[-\frac{1}{3b}; b \neq 0\right]$
- 6**  $\frac{b+1}{ab^2} - \frac{a-1}{a^2b}$   $\left[\frac{a+b}{a^2b^2}; a \neq 0 \wedge b \neq 0\right]$
- 7**  $\frac{1}{2a^2b} + \frac{2}{3ab^2}$   $\left[\frac{3b+4a}{6a^2b^2}; a \neq 0 \wedge b \neq 0\right]$
- 8**  $x + \frac{2x+1}{x-1}$   $\left[\frac{x^2+x+1}{x-1}; x \neq 1\right]$
- 9**  $\frac{a}{a-1} + \frac{2}{a-2} - \frac{1}{a-1}$   $\left[\frac{a}{a-2}; a \neq 2 \wedge a \neq 1\right]$
- 10**  $\frac{x+2}{x+1} - \frac{x-1}{x+2} - \frac{1}{x+1}$   $\left[\frac{3}{x+2}; x \neq -2 \wedge x \neq -1\right]$
- 11**  $\left(\frac{1}{2a^2} - \frac{1}{2b^2}\right) : \left(\frac{1}{a} + \frac{1}{b}\right)$   $\left[\frac{b-a}{2ab}; a \neq 0 \wedge b \neq 0 \wedge a \neq -b\right]$
- 12**  $\left(a+1 + \frac{2-2a}{a-1}\right) \cdot \frac{1}{2a}$   $\left[\frac{a-1}{2a}; a \neq 1 \wedge a \neq 0\right]$
- 13**  $\left(a - \frac{b^2}{a}\right) : \left(1 - \frac{b}{a}\right)$   $[a+b; a \neq 0 \wedge a \neq b]$
- 14**  $\left(1 - \frac{a}{4-a}\right) : \left(\frac{2}{a} - 1\right)$   $\left[\frac{2a}{4-a}; a \neq 0 \wedge a \neq 4 \wedge a \neq 2\right]$
- 15**  $\left(\frac{a^2+4}{a+4} - a\right) \cdot \frac{a+4}{1-a}$   $[4; a \neq -4 \wedge a \neq 1]$
- 16**  $\left(\frac{1}{a} + \frac{1}{a+1}\right) \cdot \left(1 - \frac{a}{2a+1}\right)$   $\left[\frac{1}{a}; a \neq 0 \wedge a \neq -1 \wedge a \neq -\frac{1}{2}\right]$
- 17**  $\left(a - \frac{a}{a+1}\right) : \left(1 - \frac{2a}{a-1}\right) \cdot \left(\frac{1}{a^2} + \frac{2}{a} + 1\right)$   $[1-a; a \neq \pm 1 \wedge a \neq 0]$
- 18**  $\left(\frac{1}{4x^2} - \frac{1}{4y^2}\right) : \left(\frac{1}{2x} + \frac{1}{2y}\right) \cdot 4xy$   $[2(y-x); x \neq 0 \wedge y \neq 0 \wedge x \neq -y]$